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FILTERING FOR PRECISION POINTING AND TRACKING WITH APPLICATION FOR AIRCRAFT TO SATELLITE TRACKING

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FILTERING FOR PRECISION POINTING AND TRACKING WITH APPLICATION FOR AIRCRAFT TO SATELLITE TRACKING

BY

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Abstract

Many aerospace problems include the requirement for precision pointing and tracking from one accelerating vehicle to another. This paper considers the use of Kalman filtering for a general class of high precision pointing and tracking applications and the application of the general framework to a specific problem. A general framework which contains all known error sources is developed for a particular Kalman filter. With a covariance sensitivity analysis, this framework can be used to determine the performance of a reduced order filter and conduct a hardware requirements analysis and trade off. In particular, the paper addresses the application of the general framework for an aircraft to satellite precision tracking problem.

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Introduction

There are many aerospace problems which include the requirement for precision pointing and tracking from one accelerating vehicle to another. If knowledge of the position and velocity states of the two vehicles is perfect, the pointing and tracking problem is straightforward. Unfortunately, such knowledge is never available. In practice, the instrumentation on board the tracking vehicle is imperfect and provides measurement information which is corrupted by noise and other unwanted effects. For example, a radar tracking device will in general be imperfect because of random phase front distortion or scintillation noise which might be target dependent and could prevent the required degree of accuracy from being achieved. In addition, the target vehicle dynamics might be unknown and must therefore be modeled as a random process. For example, when passively tracking another aircraft, the aircraft acceleration profile is unknown but could be modeled as a first order Markov process [9,10,11]. The model for a low orbit satellite could include the low order gravitational harmonics for the Earth; high order harmonics and other perturbations could then be modeled as random processes. In general, therefore, perfect knowledge of the system state is not available in the pointing and tracking problem, and stochastic estimation techniques are necessary.

This paper contains the development of a general framework for application of the extended Kalman filter to the precision pointing and tracking problem in order to estimate certain necessary physical variables in the problem. The fundamental system dynamics are described by a set of primary state equations to which are adjoined the state equations necessary to

describe instrument errors. This total state description is the "true system" model. However this model is of high order (typically 60-100 states) and the optimum filter based upon it could not therefore be implemented with a small on-board computer. On the other hand, the true system model or truth model may be used in a covariance sensitivity analysis to yield the true performance of a reduced order filter which is small enough for on-board implementation. By testing the sensitivity of approximations from the truth model to the reduced order filter, a hardware trade-off analysis will indicate instrumentation requirements. The reason for this approach along with the necessary sensitivity equations is more fully developed in reference [21]. This general approach is applied in the paper to the particular problem in which a high altitude aircraft must track a low polar orbit satellite to precise degree of accuracy. Some of the results showing the sensitivity of the tracking errors to measuring instrument precision are presented. More generally, the framework has been applied by the authors or could be applied in aircraft to aircraft, moving ground vehicle to aircraft, and satellite to satellite problems.

Work in the area of Kalman filtering for pointing and tracking includes that by Fitts [1,2,3,4] in which filtering is accomplished in the inertial reference frame. Detailed modeling of error sources was not considered. A general study by Pearson [5,6] again does not include error source modeling. Other studies in this area by Landau [7], Fitzgerald [8,9] and others [10,11,12,13] do not include error source modeling. None of the references considers the particular problem of aircraft to satellite tracking.

General Framework

The fundamental objective is to establish an accurate line of sight to the target, vehicle being tracked, along which some form of electromagnetic link will be maintained. For example, to illuminate a target with a very narrow beamwidth communications laser could require tracking accuracies in the order of microradians. Even with highly sophisticated measuring and control instruments it may not be possible to achieve such accuracy without using optimal estimation and control.

This section considers the modeling of the physical variables necessary to establish a line of sight in both position and velocity and the modeling of the instrument error sources inherent in the problem of estimation for pointing and tracking. The approach is general as it may be used for a number of applications of which only one is shown in the next section.

In this general approach it is assumed that the target is passive. That is, it does not assist the tracker in any way by providing target state information. This assumption implies that some model must be developed for the target to obtain the estimate of target acceleration which is necessary to solve the tracking problem.

Figure 1 illustrates the basic tracker geometry. The tracking device is controlled in azimuth and elevation relative to a reference coordinate system. The tracker coordinate system is shown in this figure. Note that the x_t axis aligns with the tracking device bore-sight. For perfect tracking this will lie along the line of sight between tracker and target. Figure 2 illustrates the two coordinate frames of primary interest. The tracker frame is misaligned from the line of sight frame because of imperfect

tracking. Note that this misalignment is described by two Euler angle rotations $\delta\varepsilon$ and $\delta\eta$ which are chosen in general to be about axes where misalignment measurements would in practice be available from a sensor.

The need for a target acceleration estimate to solve the tracking problem has already been indicated. To control the tracking device it is necessary to estimate the line of sight angular velocity and the tracker misalignment. These estimates provide rate and position feedback control respectively to the tracker motors. In some problems it may also be necessary to estimate range and range rate of the target. Now with the exception of target acceleration, all these parameters are line of sight and tracker parameters and since ideally the tracker frame will coincide with the line of sight frame it is logical to choose the latter frame in which to model the system. This choice has the disadvantage that target acceleration must be transformed from inertial coordinates in which estimates are most likely to be available, into tracker or line of sight coordinates. An alternative would be to model completely in the inertial frame but this would necessitate transforming the line of sight and tracker states into inertial coordinates. This paper uses the line of sight frame for modeling since this involves the least number of coordinate transformations.

The system dynamics is simply described by considering the relative position vector of the target from the tracker. Let r be the relative position vector and ω_{ls} the angular velocity vector of the line of sight relative to inertial space. Then, by the Theorem of Cariolis,

$$\left.\frac{dr}{dt}\right|_{i} = \left.\frac{dr}{dt}\right|_{ls} + \omega_{ls} \times r \tag{1}$$

where $\Big|_i$ indicates the differential is taken with respect to inertial space, etc. Similarly,

$$\left.\frac{d^2 r}{dt^2}\right|_i = \left.\frac{d^2 r}{dt^2}\right|_{1s} + 2\,\omega_{1s} \times \left.\frac{dr}{dt}\right|_{1s} + \left.\frac{d\,\omega_{1s}}{dt}\right|_{1s} + \omega_{1s} \times \omega_{1s} \times r \qquad (2)$$

Finally, the motion between the tracker coordinate frame and the line of sight coordinate frame is characterized by the following matrix differential equation [15]:

$$\dot{C}^{t}_{1s} = C^{t}_{1s} W_{1s} + W_t C^{t}_{1s} \qquad (3)$$

where C^{t}_{1s} is the coordinate transformation matrix from line of sight coordinates to tracker coordinates and W_{1s}, W_t are the line of sight and tracker angular velocity skew symmetric "cross product" matrices respectively. Equations (1), (2) and (3) are used to obtain the primary system state equations excluding target state equations as follows:

From equation (1) let R = range = $|r|$ and let V_r = range rate, then:

$$\dot{R} = V_r \qquad (4)$$

Let a_r be the relative acceleration vector of target from tracker which is equal to $a_{TARGET} - a_{TRACKER}$, ω be the angular velocity relative to inertial space, subscripts x, y, and z indicate vector components in a right hand orthogonal coordinate system, superscripts 1s and t indicate the relevant coordinate system, and the small angle approximations to be used as

$$\sin \delta\varepsilon = \delta\varepsilon,$$

$$\sin \delta\eta = \delta\eta,$$

$$\cos \delta\varepsilon = \cos \delta\eta = 1,$$

and

$$\delta\varepsilon\, \delta\eta = 0.$$

From equations (2) and (3):

$$\dot{\omega}^{1s}_{1s_y} = -\frac{1}{R} a^t_{r_z} - \frac{2\, V_r}{R} \omega^{1s}_{1s_y} + \omega^{1s}_{1s_z} \omega^t_{t_x} + \left\{ \omega^{1s}_{1s_z} \left[\delta\eta\, \omega^t_{t_y} - \delta\varepsilon\, \omega^t_{t_z} \right] - \frac{\delta\varepsilon\, a^t_{r_x}}{R} \right\} \tag{5}$$

$$\dot{\omega}^{1s}_{1s_z} = \frac{1}{R} a^t_{r_y} - \frac{2\, V_r}{R} \omega^{1s}_{1s_z} - \omega^{1s}_{1s_y} \omega^t_{t_x} - \left\{ \omega^{1s}_{1s_y} \left[\delta\eta\, \omega^t_{t_y} - \delta\varepsilon\, \omega^t_{t_z} \right] + \frac{\delta\eta\, a^t_{r_x}}{R} \right\} \tag{6}$$

$$\dot{\delta\varepsilon} = \omega^{1s}_{1s_y} - \omega^t_{t_y} + \delta\eta\, \omega^t_{t_x} \tag{7}$$

$$\dot{\delta\eta} = \omega^{1s}_{1s_z} - \omega^t_{t_z} - \delta\varepsilon\, \omega^t_{t_x} \tag{8}$$

$$\dot{V}_r = a^t_{r_x} + R\left[\left(\omega^{1s}_{1s_y}\right)^2 + \left(\omega^{1s}_{1s_z}\right)^2 \right] + \left\{ \delta\eta\, a^t_{r_y} - \delta\varepsilon\, a^t_{r_z} \right\} \tag{9}$$

Equations (4-9) are the state equations describing the range, range rate, line of sight angular velocity and tracking device misalignment. Note that the component of lines of sight angular velocity $\omega_{ls_x}^{ls}$ is not included as a state, since angular velocity along the line of sight is not required for control. However the tracker angular velocity vector ω_t^t, which is included as a system parameter, is required along all three tracker axes. The relative acceleration vector a_r is the difference between the target acceleration vector a_{tar} and the tracker acceleration vector a_t which is a system parameter. The state modeling for target acceleration will be described below. It is important to note at this point that the inclusion of system parameters such as the angular rate of the tracker implies that measurements of those parameters are available.

The modeling of the target acceleration is the most difficult part of the problem. The reader will recall that no target state information is directly available since the target is assumed generally to be passive. If the target vehicle is an aircraft, then an exponentially time correlated acceleration model, which implies that the target motion is uncertain, may be used [9,11]. Clearly, if more information about the target acceleration is available, then this should be used in the model. For example, if it is certain that the target does not maneuver, then a zero acceleration model is appropriate.

If the target is a satellite, then accurate equations of motion may be developed and utilized to give an acceleration model. An initial orbit may be given through ephemeris data. This might be no more than an initial orbit prediction necessary for satellite acquisition. The gravitational motion due to the non-spherical Earth, the Sun and the Moon can be accurately

modeled although the model may be complex. The effects of drag and solar radiation pressure are more difficult to describe since they depend on the physical characteristics of the satellite, i.e., its mass, surface area and shape. Furthermore, if the satellite is unknown then there may be a finite probability that the satellite is independently commanded to execute a ΔV maneuver or attitude change. It may, in fact, be acceptable to ignore all these effects but they must initially be included in the truth model until the filter sensitivity to them has been determined for a particular orbital condition.

Perfect knowledge of the satellite physical characteristics cannot be assumed in the general approach to the problem. It is reasonable however to assume some statistical knowledge. To describe the atmospheric drag and solar radiation pressure effects, the use of a ballistic coefficient and solar pressure coefficient is necessary. Since the satellite physical characteristics are unlikely to change with time these coefficients can reasonably be modeled as random biases with the general state equation

$$\dot{X} = 0.$$

Satellite ΔV maneuvers will most probably be of short duration unless, for example, the satellite is continually thrusting to follow a drag-free orbit. Modeling is best accomplished based on some a priori knowledge of the satellite function and adaptive techniques. Alternatively, a simpler approach is to assume a worst case size and probability of maneuver and thereby retain a large measure of uncertainty in the orbit. This safe approach can however unnecessarily degrade the ultimate tracking capability if the maneuver does not occur.

In summary, each type of target vehicle presents a different problem the physics of which must be carefully analyzed. If no information is available then the exponentially correlated random variable model [9,11] may be used. Any information which is available must be used if the best model is to be found.

The problem of measurement errors was briefly described in the introduction but the extent of this problem was not explained. When a reduced order filter is applied in the real world its performance can be highly sensitive to measurement errors. To implement a reduced order filter it is therefore necessary to conduct a full sensitivity analysis which implies that the system truth model must include all known error sources. The subsequent analysis will hopefully show that many of the errors can be ignored or discarded and what the expected filter degradation will be.

Particular applications of this general framework generate slightly different measurement requirements. The following is a description of a typical measurement set which will suit most high precision requirements. Measurements are assumed to be available for the following states and parameters:

tracker angular velocity, ω_t^t;

tracker origin inertial acceleration, a_t;

tracking device misalignment from the true line of sight, $\delta\varepsilon$, $\delta\eta$;

range, R; and

range rate, V_r.

The length of this paper prohibits a detailed description of every measurement, but to illustrate the principles, consider the measurement model of tracker velocity. The three rate gyroscopes are mounted along the three

tracker axes and yield the measurement

$$\omega_{t_{i_m}} = \omega_{t_i} + \delta k_{gi}\,\omega_{t_i} + b_{gi} + c_{gi}$$

$$+ \sum_{l=1}^{3} k_{il}\, a_{t_l} + \left[\Delta\, C_g^t\, \omega_t\right]_i + \xi_{gi} \qquad (10)$$

$$i = x, y, z.$$

Now the components of ω_t are ω_{tx}, ω_{ty} and ω_{tz} which are the true angular velocity components along the tracker x, y and z axes. The measurement vector components are ω_{txm}, ω_{tym} and ω_{tzm} respectively. The terms δk_{gi} are torque scale factor errors. During manufacture and assembly of the rate gyro, the calibration process will remove the torque scale factor error as far as possible but there will remain a small residual component which cannot be compensated for. Since such a scale factor error is unlikely to change with time, particularly in the short term, a random bias model is chosen. This model has the differential equation for the x-gyro scale factor error for example of

$$\dot{\delta k}_{gx} = 0.$$

The statistics of this factor may be found from gyro test data or from an engineering estimate. Note that the random bias mode implies that both mean $E\{\delta k_{gx}\}$ and second moment $E\{\delta k_{gx}{}^2\}$ stay constant with time.

The drift characteristic of a rate gyro can be expressed as the sum of a constant bias drift and a time varying drift. The terms b_{gi}, i = x, y, z

are bias drifts which describe those components of gyro drift which do not change in time. The time varying drifts are represented by the terms c_{gi}, $i = x, y, z$. Again, the bias terms b_{gi} are modeled using the bias random variable, for example

$$\dot{b}_{gx} = 0.$$

The model used for the c_{gi} terms would depend on the type and quality of gyro. Typically such time correlated drifts can be described by exponentially time correlated random variables with the first order differential equation form, for example

$$\dot{c}_{gx} = -\beta_{gx} c_{gx} + \sqrt{2\beta_{gx}}\, \sigma_{gx} u_{gx}$$

in which $\beta_{gx} = \frac{1}{\tau_{gx}}$ and τ_{gx} is the drift process correlation time, σ_{gx} is the rms value of the process and u_{gx} is a unity variance white driving noise. Note again however that the model requires knowledge of the drift process in terms of process correlation time and rms value.

A rate gyro is ideally insensitive to accelerations. However, in practice, a mass unbalance will exist because the center of mass and center of rotation do not coincide. The mass unbalance coefficients k_{i1} do, however, remain constant with time and can therefore again be represented with the random bias model, for example

$$\dot{k}_{xy} = 0$$

where k_{xy} is the coefficient of mass unbalance along the tracker

x-direction due to the mass unbalance effect along the tracker y-direction. If the rate gyroscope is also likely to be sensitive to products of acceleration or g^2-sensitive errors then the model should also include random bias coefficients to account for these effects.

In practice, the axes along which the rate gyroscopes are sensitive do not coincide with the true tracker coordinates. A measurement from one rate gyro will therefore include components, however small, of angular velocity along the other two tracker axes. By assuming that the displacement angles are small enough for small angle approximations to be valid, the coordinate transformation matrix can be approximated as follows. C_G^T is the coordinate transformation matrix from gyro coordinates (G) to tracker coordinat)
where

$$C_G^T = I + \Delta C_G^T$$

where

$$\Delta C_G^T = \begin{bmatrix} 0 & \theta_{yz} & \theta_{zy} \\ \theta_{xz} & 0 & \theta_{zx} \\ \theta_{xy} & \theta_{yx} & 0 \end{bmatrix}.$$

Thus the term $\left[\Delta C_G^T \omega_t\right]_i$, $i = x, y, z$ accounts for this error angle transformation and since misalignment will be constant with time the θ_{ij} terms can be described by the random bias model, for example

$$\dot{\theta}_{xy} = 0.$$

Finally, an additive white noise ξ_{gi}, $i = x, y, z$ is included in the model

to account for those higher order effects which are not otherwise modeled. Choice of variance for this noise may be difficult to determine but should be of the same order of magnitude as that of the smallest modeled effect.

At this point it is worthwhile reminding the reader that the above model includes all reasonable and common sources of error. A sensitivity analysis will indicate the dominant error terms, but note that each problem and associated measurement sensor will produce different results. Hopefully, the effect of many of the error terms in a particular application will be small relative to the required accuracy and only the dominant errors will therefore need to be included in the filter.

The following is a summary of the remaining measurement models with a brief description of each term.

Tracker Acceleration. The measurement model for tracker acceleration is

$$\begin{aligned} a_{t_{i_m}} &= a_{ti} + \delta k_{ai}\, a_{ti} + b_{ai} + c_{ai} + k_{i1}\, a_{ti}^2 \\ &\quad + k_{i2}\, a_{ti}^3 + k_{i3}\, a_{tj} + k_{i4}\, a_{tk} \\ &\quad + \left[\Delta\, C_A^N\, a_t\right]_i + \xi_{ai} \;;\; i = x,y,z \;;\; j,k \neq i \;;\; j \neq k \end{aligned} \tag{11}$$

in which $a_{t_{i_m}}$ $i = x, y, z$ are the measurements of the true tracker origin acceleration components a_{ti}, $i = x, y, z$. The accelerometer scale factor errors k_{ai} and bias errors b_{ai} are modeled again as random biases. c_{ai} are accelerometer drift errors modeled as first order exponentially correlated random variables. The coefficients k_{i1}, k_{i2}, k_{i3} and k_{i4} are non-linear calibration coefficients to account for g^2, g^3 and cross track

accelerations respectively and are modeled as random biases. C_A^N is an error angle transformation matrix relating acceleration in actual (A) accelerometer coordinates to acceleration in nominal (N) accelerometer coordinates. The elements of this matrix are small constant angles modeled as random biases. In general, the nominal accelerometer coordinate system does not align with the tracker coordinate system. If necessary, the transformation

$$a_t^T = C_N^T \, a_t^N$$

can be accomplished. Finally, ξ_{ai} is an additive white noise term to account for the remaining unmodeled errors.

Tracker Angular Misalignment $\delta\varepsilon$, $\delta\eta$ (Boresight Error)

The tracker misalignment measurement model is

$$\delta_{\varepsilon_m} = k_\varepsilon(\delta_\varepsilon + S_\varepsilon) + \delta k_\varepsilon + b_\varepsilon + \xi_\varepsilon \tag{12}$$

$$\delta_{\eta_m} = k_\eta(\delta_\eta + S_\eta) + \delta k_\eta + b_\eta + \xi_\eta \tag{13}$$

where k_ε and k_η are deterministic scale factors, S_ε and S_η are target induced scintillation errors as observed from the tracker frame. These errors can be modeled as coupled first order exponentially time-correlated random variables with equations

$$\dot{S}_\varepsilon = -\beta_\varepsilon S_\varepsilon - \omega_{tx} S_\eta + \sqrt{2\beta_\varepsilon}\,\sigma_\varepsilon u_\varepsilon \tag{14}$$

$$\dot{S}_\eta = -\beta_\eta S_\eta - \omega_{tx} S_\varepsilon + \sqrt{2\beta_\eta}\,\sigma_\eta u_\eta . \tag{15}$$

The δk_ϵ and δk_η scale factor errors and b_ϵ and b_η bias errors are modeled using the simple bias random variable model. Finally, ξ_ϵ and ξ_η are additive white noise terms to account for unmodeled effects.

Range. The range measurement model is

$$R_m = k_r (R + S_r) + k_r R + b_r + \xi_r \tag{16}$$

where k_r is a deterministic scale factor, S_r is a range scintillation error modeled as a first order exponentially correlated random variable. k_r, a scale factor error and b_r, a bias error are modeled as random biases. ξ_k is an additive white noise to account for unmodeled effects.

Range Rate. The range rate measurement model is

$$V_{r_m} \doteq k_v (V_r + S_v) + k_v V_r + b_v + \xi_{v4} \tag{17}$$

where the terms have appropriate significance as for range.

The additional state equations resulting from the above measurement models are secondary state equations which are adjoined to the primary state description to form the full state truth model. For the measurements described there are 63 secondary state equations. For the typical tracking problem therefore the total state dimension for the truth model is about 75 depending upon the number of states required to model the target acceleration. While it is doubtful that any practical problem would require the inclusion of all the above secondary state equations in the truth model, even a modest

requirement to adjoin secondary states results in a large dimension truth model which could not be handled in an on-board computational facility.

It is important to examine each measurement and decide how that information should be used. Clearly, the measurements of δ_ε, δ_η, R and V_r are direct measurements of primary system states and will therefore form a system measurement vector. The measurement of a_t however is a parameter value which is substituted into the appropriate state equations. The measurement of ω_t could be handled in an identical way. However, in this case the information can be considered as a pseudo-measurement of the angular velocity of the line of sight. The justification for this reasoning is that the tracking device will ideally follow the line of sight in such a way that the mean angular velocity deviation between ω_{ls} and ω_t is zero.

So far, the system truth model state and measurement equations have been developed. Two problems are apparent; the equations are in general nonlinear and the vector dimensions are too great for the optimal filter to be implemented on board the tracking vehicle. The nonlinearity of the equations is overcome by linearizing so that the equations for the extended Kalman filter are valid. It is necessary however to implement a filter on board the tracking vehicle. A reduced order (dimension) filter must therefore be designed. In general the performance of a reduced order filter will be sub-optimal and must be evaluated against the theoretically optimal performance of a filter based on the full truth state description. This evaluation is carried out initially using the method of covariance sensitivity analysis described by Asher and Reeves [19]. However, it can also be shown [20] that the reduced order filter is in general conditionally biased and to obtain all the statistical information the results of reference [20] must be applied.

The process of designing a reduced order filter using the covariance sensitivity analysis will not only indicate which errors and perturbations can be ignored but will also show the degree of precision necessary to achieve a specified tracking accuracy with a particular reduced order filter. There is currently no mathematically precise algorithm by which such a design process can be conducted, but the flow chart in Figure 3 shows the basic method applied. The filter should also be simulated via Monte Carlo methods.

An initial intuitive choice of reduced order filter parameters and states is made. For example, the gyro measurement might be assumed to take the simple form

$$\omega_{tim} = \omega_{ti} + c_{gi} + \xi_{gi}$$

where the predominant drift term has been retained but the remaining terms have been accounted for by increasing the variance of the additional white noise ξ_{gi}. For another example, a satellite orbit may be assumed to be a simple two body orbit in which the gravitational harmonics and other perturbations are modeled by a simple additive white noise of suitable variance. With this basis, the covariance sensitivity analysis is used to tune the filter by adjusting the reduced order filter parameters so that the estimation error and filter sensitivity are minimized. Note that this will be an iterative procedure but no changes are made to the system truth model at this time.

The tuning process will also indicate where the filter is most sensitive to measurement error sources. An examination of this sensitivity is then made and areas for possible hardware changes or trade-offs are identified.

For example, it may be apparent that specifications on gyro drift could be relaxed but angle track scintillation errors must be reduced. The system is therefore redesigned and changes are made to both the full state optimal filter and the reduced order sub-optimal filter before the tuning process is repeated.

Application in Aircraft-to-Satellite Tracking

To illustrate the application of the general framework described above, consider the problem in which a high altitude aircraft is tracking a low polar-orbit satellite. Assume the required accuracy is arbitrarily taken to be on the order of microradians. Applying a covariance sensitivity analysis will show first, which states must be retained in a reduced order filter, and secondly, what the hardware requirements might be in order to meet or improve on the accuracy requirement.

The system truth model comprises the state and measurement equations described in the general framework together with a set of target state equations which describe the motion of the satellite. The target equations are

$$\dot{x}_s = v_s \tag{18}$$

$$\dot{v}_s = a_g + a_d + a_s + a_m + a_p + \xi_s \tag{19}$$

$$\dot{B} = 0 \tag{20}$$

$$\dot{S} = 0 \tag{21}$$

where x_s and v_s are the satellite inertial position and velocity vectors. The gravitational force due to the Earth is described by the vector a_g

and for this problem includes harmonics up to sixth order. The atmospheric drag force is described by the vector a_d which is dependent on a satellite ballistic coefficient B. The perturbational forces due to the sun and moon are described by the vectors a_s and a_m respectively and the solar pressure force is described by the vector a_p which is dependent on a solar pressure coefficient S. Finally, the unmodeled effects of higher order gravitational harmonics and other perturbations are accounted for by the additive noise vector ξ_s which is assumed to be a zero mean white noise. Assuming that the satellite is passive (but not cooperative), the probability that the satellite executes any ΔV maneuver or attitude adjustment is zero. The physical properties of the satellite such as mass shape, size and surface area are assumed to remain constant for the duration of tracking. Thus the best physical knowledge available for the satellite is assumed to be a mean and standard deviation to describe an expected distribution of these satellite physical properties. With this basis, the two coefficients B and S are chosen to be random biases with the state description given by equations (20) and (21).

The resulting truth model for this application has a state vector dimension of 61, a measurement vector dimension of 5 and a parameter measurement vector for tracker angular velocity and tracker origin acceleration of dimension 6. The accelerometer crosstrack errors were assumed small enough to be neglected. The accelerometer and gyro bias terms were included into the initial conditions for the respective correlated drift terms. A reduced order filter is chosen by making the following simplifications from the truth model.

a. Satellite motion can be described by a simple two-body orbit perturbed by a white driving noise vector. The variance of this vector (ξ_s in the truth model) is increased to account for the two-body approximations.

b. The equations describing line of sight angular velocity and range rate are simplified to remove the terms (see equations 5, 6 and 9) which account for the effect of tracker boresight error. In each equation, a white driving noise of appropriate variance is introduced to account for the approximation.

c. In each measurement equation, all sources of error are removed and the variance of the white driving noise is increased in each case to compensate for the approximation.

The following set of state and measurement equations results from the first attempt at a reduced order filter.

State Equations

$$\left.\begin{aligned} \dot{x}_s &= v_s && (22) \\ \dot{v}_s &= \frac{-\mu_\oplus x_s}{|x_s|^3} + \xi_s && (23) \end{aligned}\right\} \text{Satellite motion}$$

$$\left.\begin{aligned} \dot{\omega}^{ls}_{ls_y} &= -\frac{1}{R} a^t_{r_z} - \frac{2V_r}{R}\omega^{ls}_{ls_y} + \omega^{ls}_{ls_z}\omega^t_{tx} + \xi_{ls_y} && (24) \\ \dot{\omega}^{ls}_{ls_z} &= \frac{1}{R} a^t_{r_y} - \frac{2V_r}{R}\omega^{ls}_{ls_z} - \omega^{ls}_{ls_y}\omega^t_{tx} + \xi_{ls_z} && (25) \end{aligned}\right\} \text{Line of sight angular velocity}$$

$$\left.\begin{aligned} \dot{\delta}_{\varepsilon} &= \omega^{ls}_{ls_y} - \omega^{t}_{ty} + \delta_{\eta}\,\omega^{t}_{t_x} && (26) \\ \dot{\delta}_{\eta} &= \omega^{ls}_{ls_z} - \omega^{t}_{tz} - \delta_{\varepsilon}\,\omega^{t}_{tx} && (27) \end{aligned}\right\}\text{Tracker misalignment}$$

$$\left.\begin{aligned} \dot{R} &= V_r && (28) \\ \dot{V}_r &= a^{t}_{r_x} + R\left[\left(\omega^{ls}_{ls_y}\right)^2 + \left(\omega^{ls}_{ls_z}\right)^2\right] && (29) \end{aligned}\right\}\text{Range/ Range rate}$$

Measurement Equations

$$\omega_{ti_m} = \omega_{ti} + \xi_{gi} \tag{30}$$

$$a_{ti_m} = a_{ti} + \xi_{ai} \tag{31}$$

$$\delta_{\varepsilon_m} = \delta_{\varepsilon} + \xi_{\varepsilon} \tag{32}$$

$$\delta_{\eta_m} = \delta_{\eta} + \xi_{\eta} \tag{33}$$

$$R_m = R + \xi_R \tag{34}$$

$$V_{r_m} = V_r + \xi_{V_r} \tag{35}$$

The objective of tuning a reduced order filter using the covariance sensitivity analysis technique is to force the reduced order filter error covariance to track the true full state filter error covariance as closely as possible. In practice this is achieved by adjusting the various filter noise parameters while maintaining the truth model parameters fixed and is a long process. If the reduced order filter has been over-simplified and significant error sources are not modeled, then the filter may be extremely

difficult to tune or more likely will show a divergent performance characteristic.

Consider then the filter described above. For the particular orbit chosen and the initial measuring instrument parameter set, several changes are necessary before the filter can be tuned. Because of the low orbit profile, atmospheric drag produces a significant perturbation from the two-body trajectory and cannot be properly accounted for with white noise. The drag vector a_d must therefore be re-introduced although it is sufficient to assume a fixed value for the ballistic coefficient B. It is also necessary to re-introduce the two states describing target-induced angle track scintillation to obtain acceptable reduced order filter performance. In this case, the target-induced angle track scintillation noise is assumed to have a 2-second correlation time. Because this error has a significant steady state standard deviation compared to other error sources in the angle track measurement, it is not possible to account for it in the additive white noise term. The covariance analysis for the initial choice of reduced order filter therefore results in an increased filter state dimension from 12 states to 14.

To better illustrate this tuning process, see Figures 3 and 4. Figure 3 shows the standard deviations (1 - sigma) of the tracker misalignment angle δ_ε error predicted by the reduced order filter. The filter apparently performs well and a steady state standard deviation of approximately 10 μrad results. Without carrying out a covariance analysis the engineer might consider this to be very satisfactory. However, Figure 4 is the result of the covariance analysis and shows the true standard deviation of δ_ε

committed by the particular choice of reduced order filter which produced Figure 3. In fact, as already explained, it is necessary to re-introduce the angle track scintillation error in the reduced order filter before satisfactory performance can be achieved. Figures 5 and 6 on the other hand illustrate much better tuning and in this case range error standard deviation is shown. The filter is slightly over-estimating the true error.

The next stage in the sensitivity analysis is to find the error sources included in the truth model which have the most significant effect on tracking accuracy. For this problem and an assumed parameter set, it is clear from results that rate gyro drift and angle track scintillation are the two predominant error sources. To illustrate this insight, consider the remaining measurements first. A significant error standard deviation in range measurement scintillation, bias and noise can be tolerated before angle track accuracy begins to suffer. Range rate measurement is found to be superfluous provided range information is available at 2 sec or smaller intervals. The accelerometers can be of average precision capable of measuring to within about 1% of true acceleration. As rate gyro drift is improved however, the tracking error standard deviations improve considerably, but sensitivity diminishes until improvements beyond an error standard deviation of about 0.5×10^{-6} rad/sec have little effec . In fact, the rate gyros could theoretically be perfect and no further improvement would result because the tracking accuracy is ultimately limited by angle track scintillation noise. Figure 7 shows a family of sensitivity curves for tracking accuracy against gyro drift and angle track scintillation.

This analysis is based upon an assumed nominal trajectory which is used to evaluate the system matrices. This is an approximate procedure that is valid for small errors and is exact only for the case of linear dynamics. However, the procedure is extensively and successfully used for filter analysis.

The problem is clearly complex and cannot be solved by intuition alone. In the above simplified description of a sensitivity analysis such improvement in an error source implies a hardware change. The hardware change correspondingly results in a truth model parameter change which carries the penalty of re-tuning or re-designing the reduced order filter. Moreover, as the sensitivity analysis proceeds and truth model adjustments (with hardware implications) are made, the sensitivity characteristic changes. For example, if in this problem the rate gyro and angle track measurements are both perfect, the tracking accuracy improves but becomes sensitive to other measurements and states such as range, acceleration and the satellite orbital estimate. An error budget may be used to show the relative benefit in overall performance of changing one sensor versus another, so that cost effective hardware decisions may be made.

Figure 8 shows a flow chart for a typical filter performance evaluation. The inner loop illustrates the reduced order filter tuning and re-designing to achieve satisfactory performance against a specific truth model. The outer loop illustrates the hardware requirement and trade-off process through which the system truth model is adjusted to reflect changes in hardware.

Conclusions

A general framework has been developed for the application of estimation techniques to the precision pointing and tracking problem from one accelerating vehicle to another. Modeling is carried out in the line of sight coordinate system in which measurement information is most likely to be available. There is no general model for target motion but it can be assumed that target motion will most easily be described in inertial coordinates. Specific problems with differing targets will require different target models.

The problem of handling large computational requirements has been identified and hence the need to find a reduced order filter and tune this filter against a full state system truth model. Furthermore, the method of identifying hardware requirements and trade-offs to meet specific performance criteria using the covariance sensitivity analysis has been described.

Finally, the general framework has been applied to the specific problem of tracking a satellite from an aircraft to an arbitrary degree of precision. The particular problem involved a high altitude aircraft and a low polar orbit satellite. Some results following from the covariance sensitivity analysis for this problem have been presented.

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Appendix A

Gravitational Potential Model

The model for the gravitational potential is

$$U = \frac{k_e^2 m}{r}\left[1 + \sum_{k=2}^{6}\left(\sum_{m=0}^{k} \frac{P_k^{(m)}\sin\phi}{r^k}\left\{C_{k,m}\cos(m_{\lambda_E}) + S_{k,m}\sin(m_{\lambda_E})\right\}\right)\right]$$

where k_e is the gravitational constant for the earth, m is the mass of the earth, r is the radial distance of the body from the earth center, $P_k^{(m)}$ are Legendre functions, λ_E is the longitude of the satellite with respect to the Greenwich medium, and $C_{k,m}$ and $S_{k,m}$ are harmonic coefficients for the potential model.

Drag Force

The model for the drag is

$$A_d = -\frac{1}{2}\rho B V_a \dot{r}_a$$

where p is the atmospheric density, assumed exponential, B is the vehicle ballistic coefficients, V_a is the magnitude of vehicle velocity relative to the rotating atmosphere, and $\dot{r}_a$ is the velocity vector of the vehicle relative to the rotating atmosphere.

Solar Pressure

The model for the solar pressure is

$$A_p = -KS \frac{(\cdot)_{vs}}{r_{vs}}$$

where $(\cdot)_{vs}$ is the coordinate of the sum relative to the vehicle, K is a proportionality constant, r_{vs} is the distance from the sun to the vehicle, and S is the solar pressure coefficient.

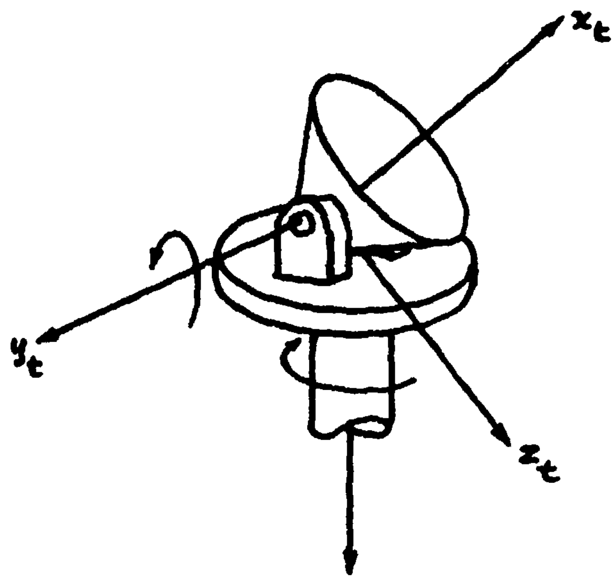

Fig 1. Basic Tracker Geometry

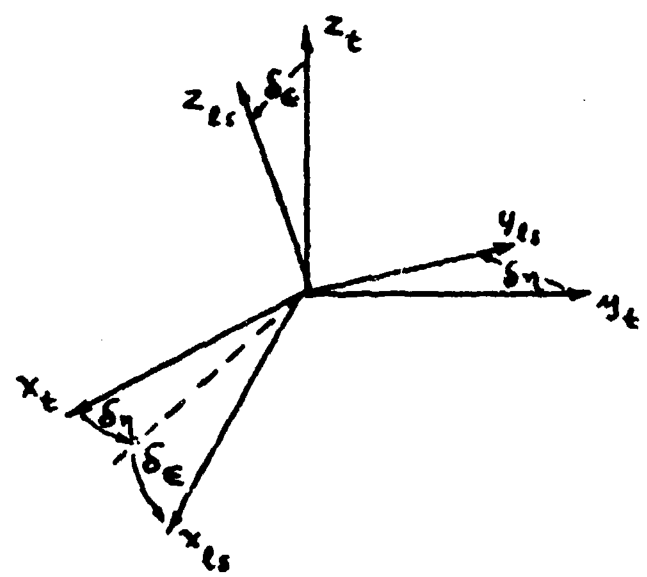

Fig 2. Tracker and Line of Sight Coordinate Frames

Tracker Misalig. ment δ_ε
Error Standard Deviation
x 10^{-3} rad

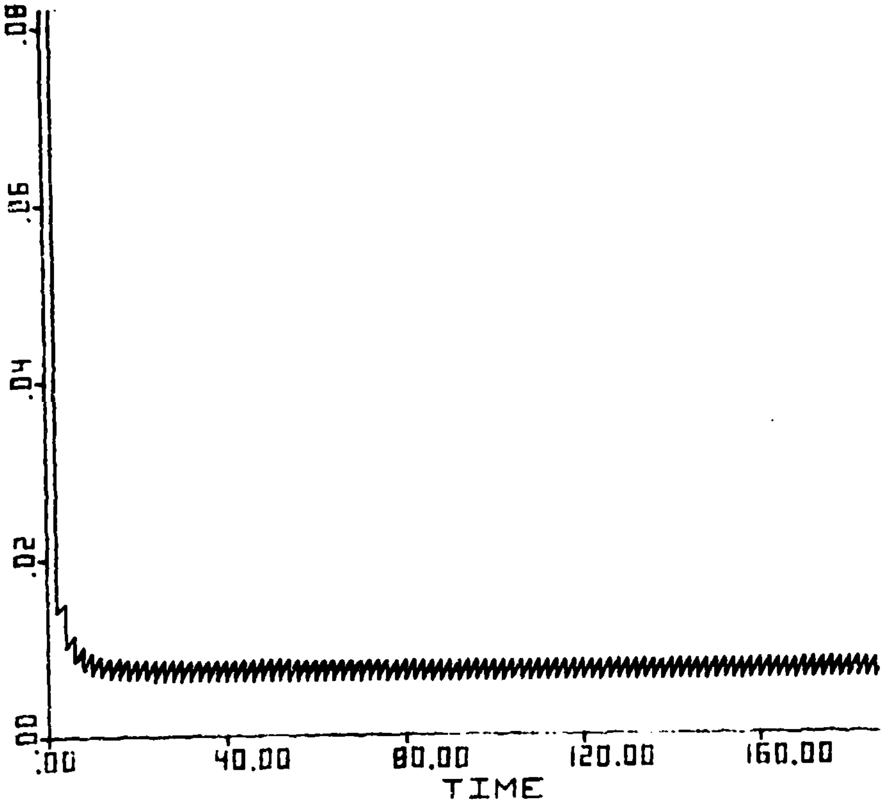

Fig 3. Reduced order filter predicted performance - δ_ε

Tracker Misalignment δ_ε
Error Standard Deviation
x 10^{-3} rad

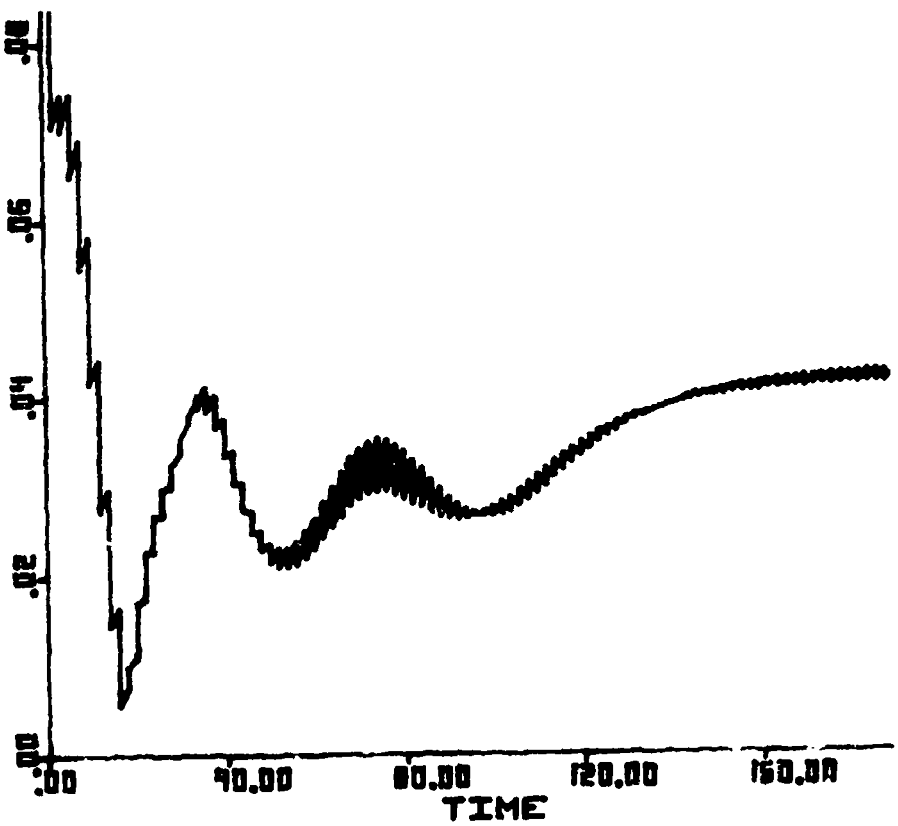

Fig 4. True reduced order filter performance - δ_ε

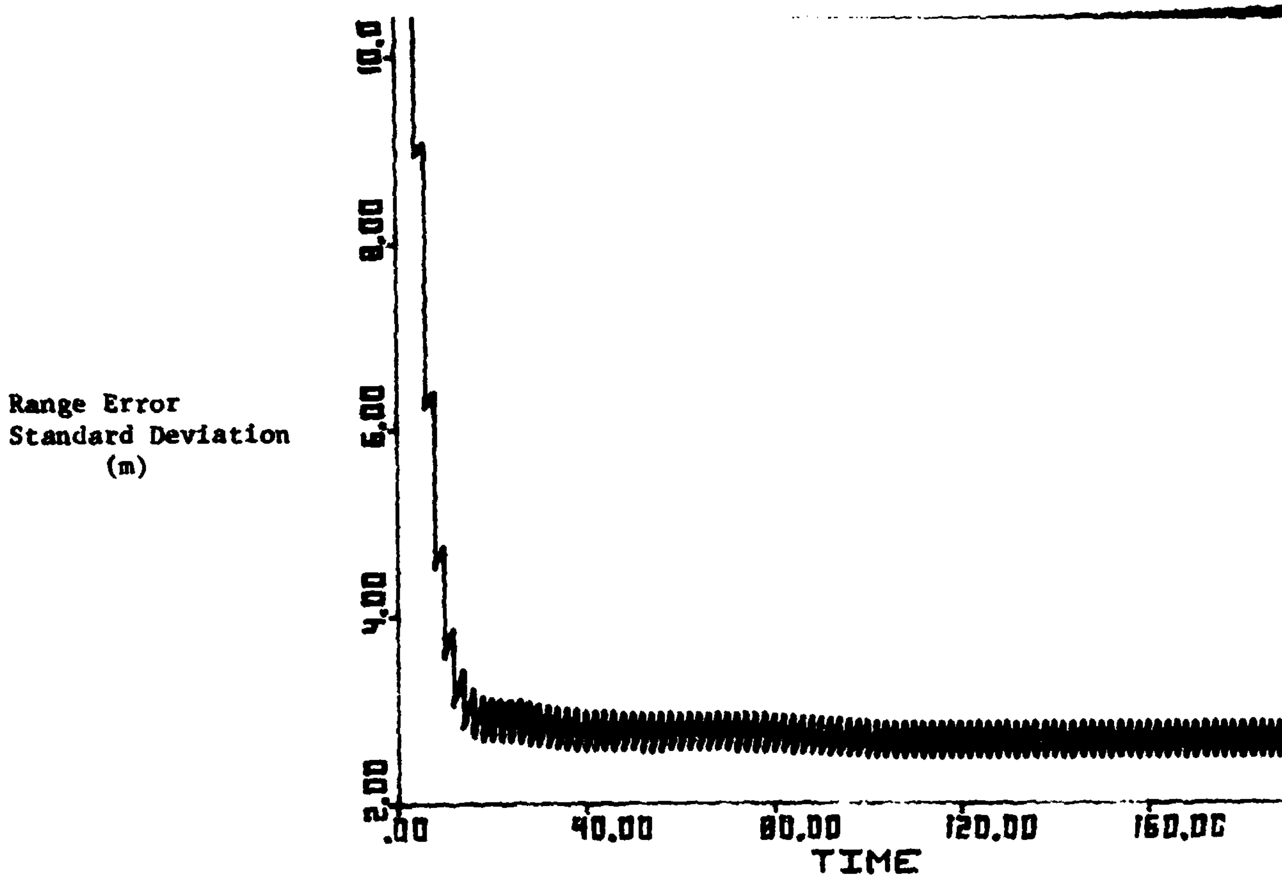

Fig 5. Reduced order filter predicted performance - Range.

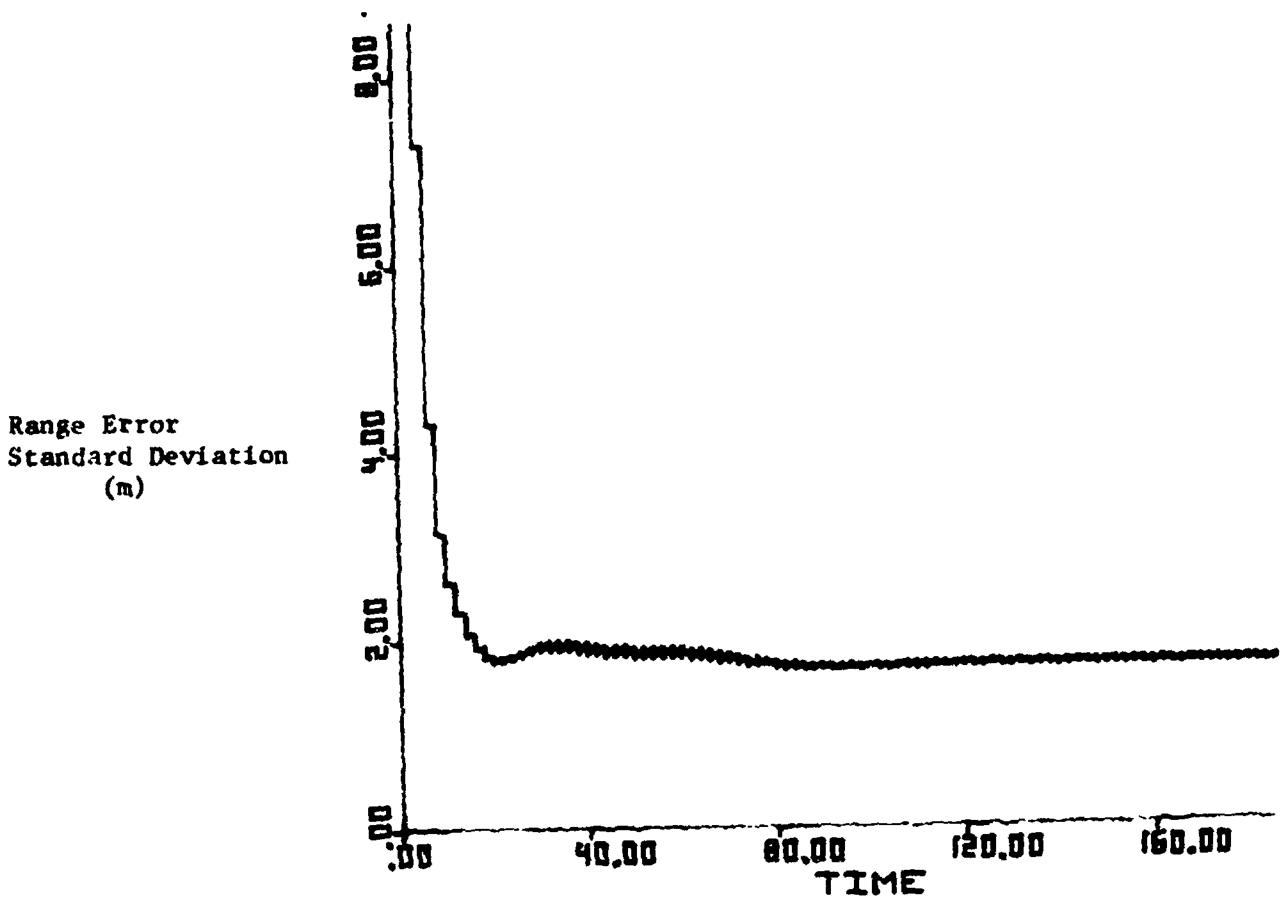

Fig 6. True reduced order filter performance - Range.

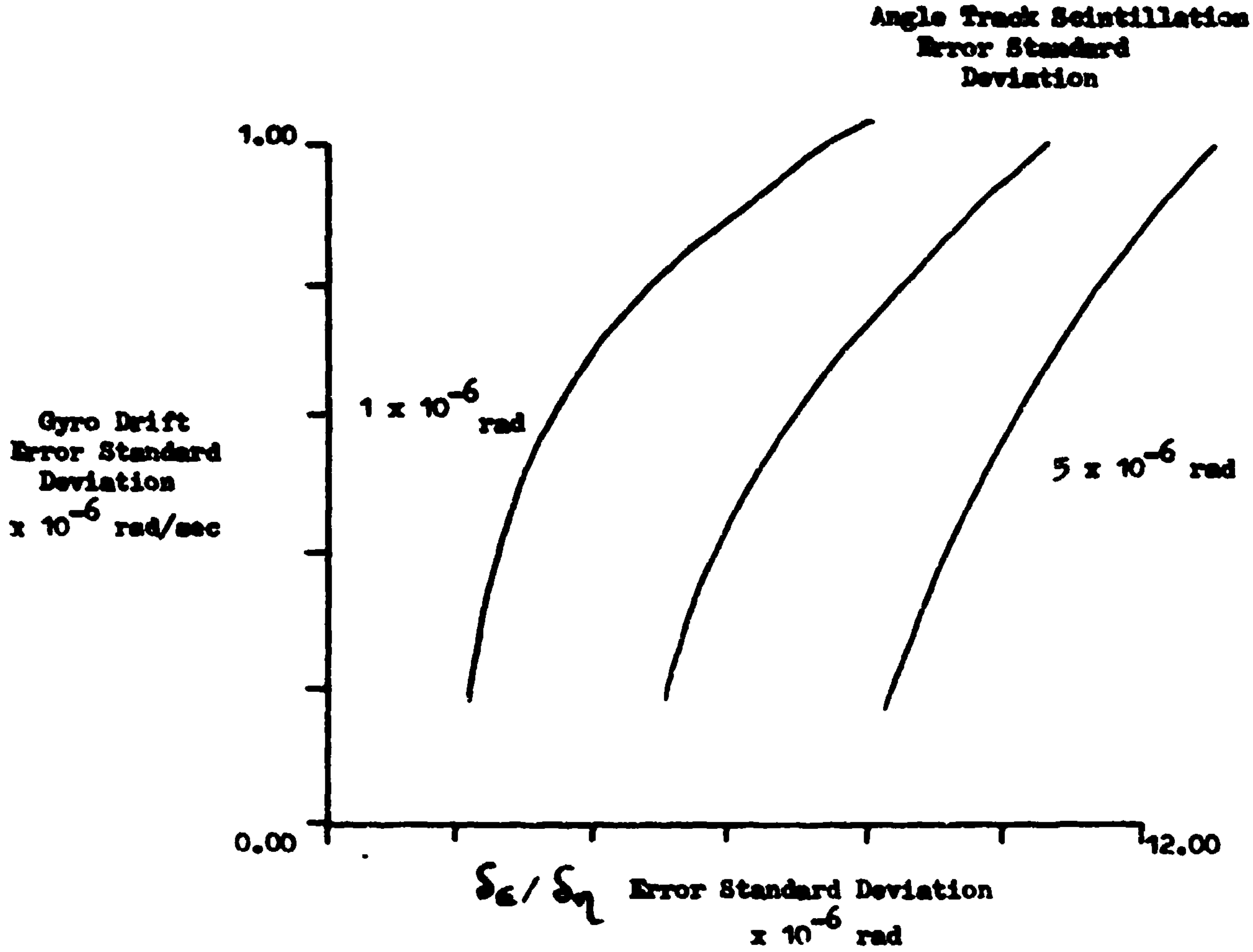

Fig 7. Tracking Accuracy against Gyro Drift and Angle Track Scintillation.

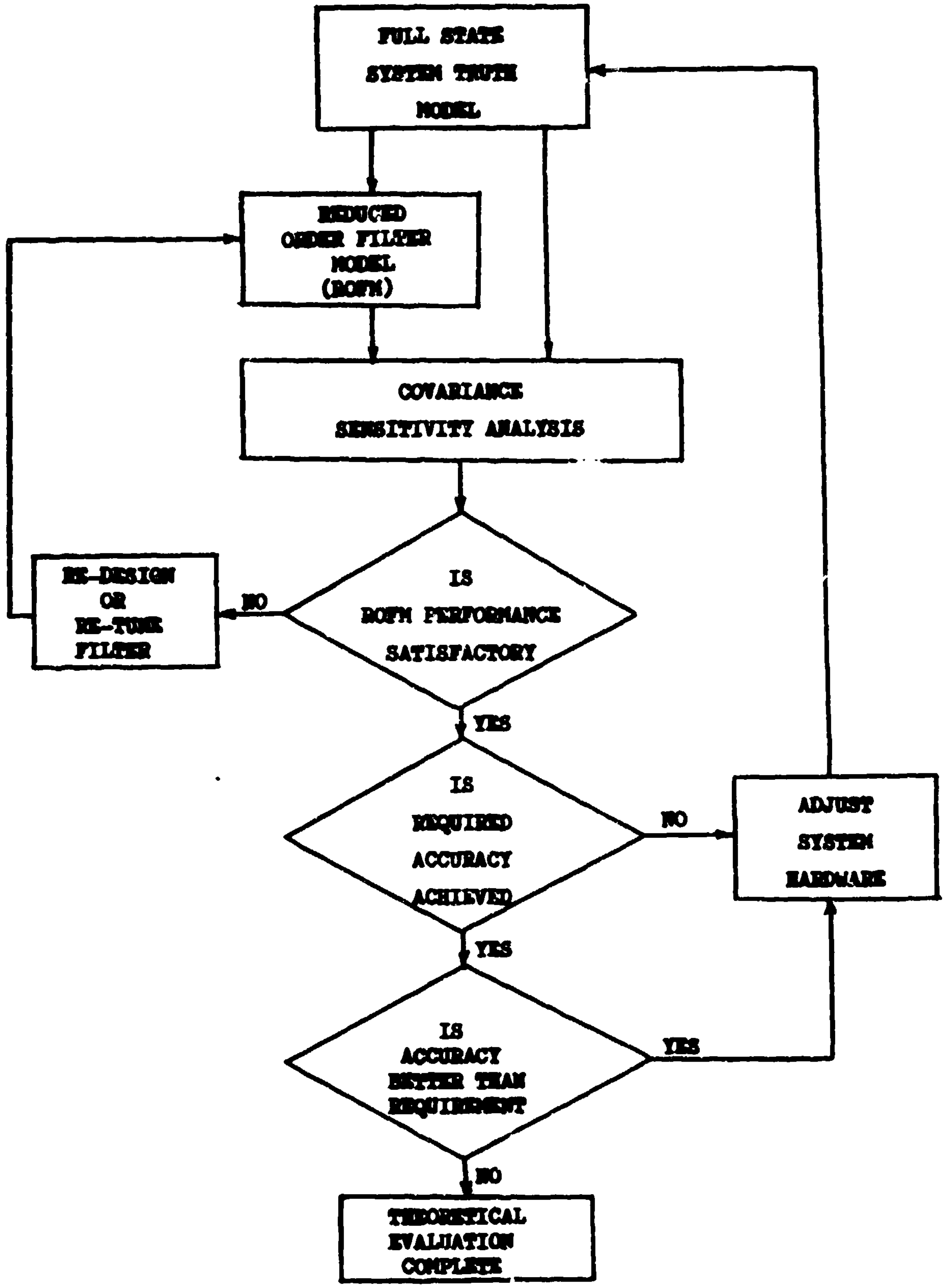

Fig 8. Filter Performance Evaluation Flow Chart